

## Solution 1 – 21 points

We will use the following 3-leveled structure to create the router:

- Level 1:  $N$  inputs
- Level 2:  $K$  intermediary nodes
- Level 3:  $N$  outputs

We group the  $N$  inputs evenly into  $K$  groups of size  $N / K$ . Nodes from each group will be connected with each of the corresponding  $K$  nodes from level 2. Each node from level 2 will be connected with each node from level 3.

This grouping produces  $N + N \cdot K$  connections. The maximum power will be achieved in the intermediary nodes, and is equal to  $N^2 / K$  ( $N$  output nodes connected with  $N / K$  inputs). We can then check for all possible values for  $K$  which one produces a solution which satisfies the power and connection constraints.

We find that a suitable value for  $K$  is approximately  $N^{(1/2)}$ . This produces  $N + N^{(3/2)}$  connections and  $N^{(3/2)}$  maximum power.

## Solution 2 – 47 points

We will use the following 4-leveled structure to create the router:

- Level 1:  $N$  inputs
- Level 2:  $K$  intermediary nodes
- Level 3:  $K$  intermediary nodes
- Level 4:  $N$  outputs

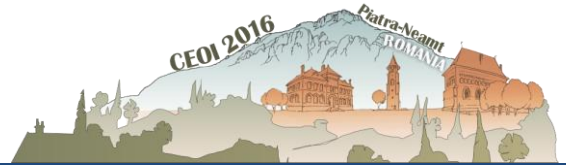
We group the  $N$  inputs evenly into  $K$  groups of size  $N / K$ . Nodes from each of the  $K$  groups will be connected with each of the corresponding  $K$  nodes from level 2. Similarly, all outputs will be grouped and connected to the corresponding nodes from level 3. Finally, each node from level 2 will be connected with each node from level 3.

This grouping produces  $2 \cdot N + K^2$  connections. The maximum power will be achieved in the intermediary nodes, and is equal to  $N^2 / K$  ( $N$  input nodes connected with  $N / K$  outputs, or  $N$  output nodes connected with  $N / K$  inputs). We can then check for all possible values for  $K$  which one produces a solution which satisfies the power and connection constraints.

We find that a suitable value for  $K$  is approximately  $N^{(2/3)}$ . This produces  $2 \cdot N + N^{(4/3)}$  connections and  $N^{(4/3)}$  maximum power.

## PROBLEM 3 – Router

DAY 2 TASK 3  
ENGLISH



### Solution 3 – 77 points

We will use the divide and conquer approach. Let's assume we want to connect the input nodes  $\{In_0, \dots, In_{N-1}\}$  to the output nodes  $\{Out_0, \dots, Out_{N-1}\}$ .

If  $n = 1$ , then we simply add an edge from  $In_0$  to  $Out_0$  and we are done. Otherwise, we will create a middle layer with nodes  $\{NextIn_0, \dots, NextIn_{N-1}\}$  and we will split this layer into two subsets,  $Next_0 = \{NextIn_0, \dots, NextIn_{\lfloor N/2 \rfloor - 1}\}$  and  $Next_1 = \{NextIn_{\lfloor N/2 \rfloor}, \dots, NextIn_{N-1}\}$ .

Then, we connect every node from  $Next_0$  to the output nodes  $\{Out_0, \dots, Out_{\lfloor N/2 \rfloor - 1}\}$  and every node from  $Next_1$  to the output nodes  $\{Out_{\lfloor N/2 \rfloor}, \dots, Out_{N-1}\}$  using the same recursive approach. Now, it is sufficient to connect every input node  $\{In_0, \dots, In_{N-1}\}$  to at least one node from  $Next_0$  and at least one node from  $Next_1$  in order to assure that every input node is connected to every output node.

To do this, we will connect  $In_i$  to  $NextIn_i$  and  $NextIn_{N/2 + (i \bmod (N - \lfloor N/2 \rfloor))}$  for  $0 \leq i < N/2$  and we will connect  $In_i$  to  $NextIn_i$  and  $NextIn_{(i - \lfloor N/2 \rfloor) \bmod \lfloor N/2 \rfloor}$  for  $N/2 \leq i < n$ .

Let  $E(n)$  be the number of edges used to connect  $n$  input nodes to  $n$  output nodes. The recurrence for this number is  $E(1) = 1$ ,  $E(N) = E(\lfloor N/2 \rfloor) + E(N - \lfloor N/2 \rfloor) + 2N$ . We can solve this even programmatically and we find  $E(N) = 2N * \log_2 N + N$ .

Let us analyze the maximum cost of a node if we connect  $n$  input nodes to  $n$  output nodes. Every input node and every output node will have a cost of exactly  $N$ . Every node at the  $k$ -th intermediate layer will be connected to  $N/2^k$  output nodes and there will be less than  $2^{k+1}$  input nodes connected to it. Thus, the maximum cost is less than  $2N$ .

### Solution 4 – 100 points

We will optimize the previous solution, in order to balance the number of edges with the maximum cost of a node.

If we try to connect  $N$  input nodes to  $N$  output nodes, we will create  $\lfloor N/t \rfloor$  input buckets and  $\lfloor N/t \rfloor$  output buckets. We will create an intermediate node for every bucket and we will connect every node  $i$  to the  $\lfloor i/t \rfloor$ -th corresponding intermediate node ( $0 \leq i < n$ ). Thus, we have two middle input and output layers with  $\lfloor N/t \rfloor$  nodes each, which we will connect using the divide and conquer approach.

This solution decreases the number of edges with a factor comparable to  $t$  and increases the maximum cost of a node with a factor comparable to  $t^2$ . The optimal value of  $t$  can be determined using various techniques, even through manual experimentation. A reasonable value for this threshold is  $t = 4$ .