

**Input File:** register.in  
**Output File:** register.out  
**Source Code:** register.pas/.c/.cpp

**100 Points**  
**Time Limit:** 1.5 s  
**Memory Limit:** 16 MB

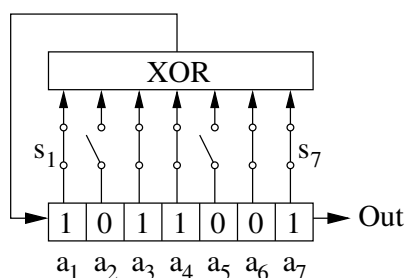
## Shift Register

A register of a computer stores  $N$  bits for computation. A shift register is a special kind of register, with bit values that can be easily shifted by one position.

Using a feedback shift register, binary pseudo-random numbers can be generated in the following way: A shift register of size  $N$  is initially filled with the bit values  $a_1, a_2, \dots, a_N$ . At each clock tick, the register outputs the value of the rightmost bit,  $a_N$ . The other bit values are shifted by one position to the right. The first position is assigned a new value  $a'_1$  as follows:

Each bit of the register is connected to an XOR gate via a switch (see figure below). For each bit  $i$  there is a switch  $s_i$  (which can be 1 or 0) that determines whether the bit value  $a_i$  is forwarded or not to the XOR gate. Let  $k_i = s_i \cdot a_i$ . The new value  $a'_1$  is set to the output value of the XOR gate,  $\text{XOR}(k_1, \dots, k_N)$ . (Remark: If the number of ones in  $k_1, \dots, k_N$  is odd, the value of  $\text{XOR}(k_1, \dots, k_N)$  is 1, else 0). Below are the formal definitions:

$$\begin{aligned} a'_1 &= \text{XOR}(k_1, \dots, k_N) \\ a'_i &= a_{i-1} \text{ for } 2 \leq i \leq N \\ \text{output} &= a_N \end{aligned}$$



tick	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	output
0	1	0	1	1	0	0	1	-
1	0	1	0	1	1	0	0	1
2	1	0	1	0	1	1	0	0
3	1	1	0	1	0	1	1	0
4	0	1	1	0	1	0	1	1
5	0	0	1	1	0	1	0	1
6	1	0	0	1	1	0	1	0
7	1	1	0	0	1	1	0	1
8	0	1	1	0	0	1	1	0
9	1	0	1	1	0	0	1	1
10	0	1	0	1	1	0	0	1
11	1	0	1	0	1	1	0	0
12	1	1	0	1	0	1	1	0
13	0	1	1	0	1	0	1	1
14	0	0	1	1	0	1	0	1

In the example above, the value  $a_1$  at tick 1 is calculated as follows:  
 $\text{XOR}(1 \cdot 1, 0 \cdot 0, 1 \cdot 1, 1 \cdot 1, 0 \cdot 0, 1 \cdot 0, 1 \cdot 1) = 0$ .

You are given the first  $2N$  output values of such a feedback shift register. From those values, you shall try to determine the switch values  $s_i$ .



# CENTRAL EUROPEAN OLYMPIAD IN INFORMATICS

Münster, Germany  
July 5-12, 2003

Page 2 of 2

English

Day 2: **register**

## Input

The first line of the input file `register.in` contains the size  $N$  of the shift register ( $1 \leq N \leq 750$ ). The second line contains  $2N$  numbers 0 or 1, which are the first  $2N$  output bit values of the shift register.

## Output

The output file `register.out` consists of exactly one line. If there is a switch setting that produces the given register output values, output the switch values  $s_i$  of any such switch setting, starting with  $s_1$ . If there are no such switch settings, output the number -1 only.

## Examples

<code>register.in</code>	<code>register.out</code>
7 1 0 0 1 1 0 1 0 1 1 0 0 1 1	1 0 1 1 0 1 1

<code>register.in</code>	<code>register.out</code>
3 0 0 0 1 1 1	-1